International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 5 Issue 7 ǁ July. 2017 ǁ PP. 06-13

A New Approach on Proportional Fuzzy Likelihood Ratio orderings of Triangular Fuzzy Random Variables

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ABSTRACT: In this chapter, we introduce a new approach on the concept of proportional fuzzy likelihood ratio orderings, increasing and decreasing proportional fuzzy likelihood ratio orderings of triangular fuzzy random variables are presented. Based on these orderings, some theorems are also established.

I. INTRODUCTION

Likelihood ratio ordering established in stochastic processes is very useful in developing bounds and approximation for performance measures of stochastic systems. Two triangular fuzzy random variables R and T with parameters means μ_1, μ_2 and standard deviationsσ_1, σ_2 respectively, are ordered in the sense of likelihood ratio ordering, when the ratio $\frac{P\{(R_\alpha^L-(\alpha-1)\,\sigma_2-\mu_2) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2-\mu_2) \leq 0\}}{P\{(T_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \ \vee (T_\alpha^U + (\alpha-1)\sigma_2-\mu_2) \leq 0\}}$ of their probability density functions is also an increasing function.

In this chapter, we discuss about the Proportional fuzzy likelihood ratio orderings of triangular fuzzy random variables based on Kwakernaak's [2] fuzzy random variables. The association of the paper is as follows. In section 2, briefly mention the α-cut of the triangular fuzzy random variables with parameters mean μ and standard deviation σ and some new definitions of the proportional, increasing and decreasing proportional fuzzy likelihood ratio orderings are presented. In section 3, we prove some theorems of proportional, increasing and decreasing proportional fuzzy likelihood ratio orderings are proved.

1.1Preliminaries

In this section some well - known basic definitions and notions will be discussed.

Definition:1.2.1
A fuzzy set A on the universal set X is defined as the set ordered pairs
$A = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0,1]\}$, where $y= \mu_A(x)$ is its membership function.

Definition:1.2.2
The support of fuzzy set A is the set of all points x in X such that$\mu_A(x)>0$.
That is, Support (A) = $\{x \in X / \mu_A(x) > 0\}$.

Definition: 1.2.3
The α-cut of α-level set of fuzzy set A is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. (i.e.,) $A_\alpha = \{X / \mu_A(x) \geq \alpha\}$

Definition: 3.2.4
A fuzzy set A on R must possess at least the following three properties to qualify as a fuzzy number.

i. A must be a normal fuzzy set
ii. A_α must be closed interval for every $\alpha \in [0,1]$
iii. The support of A, ^{0+}A must be bounded.

Among the various shapes of fuzzy numbers, triangular fuzzy number (TFN) is the most popular one. A TFN is defined as follows:

Definition: 1.2.5

Atriangular fuzzy number A = (a_1, a_2, a_3) with a_1 ,a_2, $a_3 \in R$, $a_1 \leq a_2 \leq a_3$, is a fuzzy number with membership function

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \leq x < a_2 \\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2 \leq x < a_3 \\ 0 & for\ x \geq a_3 \end{cases}$$

It is easy to check that the α-cut of a TFN A = (a_1, a_2, a_3) is of the form $A_\alpha = [a_1^\alpha, a_3^\alpha]$ with

$a_1^\alpha = (a_2 - a_1)\,\alpha + a_1, a_3^\alpha = -(a_3 - a_2)\alpha + a_3$

Definition: 3.2.6

If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1, σ_2 respectively. Then R is said to be fuzzy likelihood ratio order of triangular fuzzy number (LRO) which is less than (or) equal to T, if whenever $s \leq t$ and $u \leq v$.

Here,$s = \mu_1 - \sigma_1$,$t = \mu_2 - \sigma_2$, $u = \mu_1 + \sigma_1$and$v = \mu_2 + \sigma_2$.

$$P\,\{(R_\alpha^L - (\alpha-1)\sigma_2 - \mu_2) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \leq 0\}$$
$$P\,\{(T_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \geq 0 \vee (T_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \leq 0\}$$
$$\leq\ P\,\{(R_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \leq 0\}$$
$$P\,\{(T_\alpha^L - (\alpha-1)\sigma_2 - \mu_2) \geq 0 \vee (T_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \leq 0\}$$

It can also be written as

$$\frac{P\{(T_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0\ \vee (T_\alpha^U + (\alpha-1)\sigma_1-\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(T_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0\ \vee (T_\alpha^U\ +\ (\alpha-1)\sigma_2-\mu_2) \leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\ \sigma_2-\mu_2) \geq 0 \vee (R_\alpha^U\ +\ (\alpha-1)\sigma_2-\mu_2) \leq 0\}}$$

We will write $R \leq^{LRO} T$.

Definition: 1.2.7

If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1, σ_2 respectively, and T has a log – concave function of the triangular fuzzy random variable. Then R is said to be log – concave function of fuzzy likelihood ratio order of triangular fuzzy number (LCFLR), which is less than (or) equal to T. if whenever $s \leq t$ and$u \leq v$.

Here, $s = \mu_1 - \sigma_1$, $t = \mu_2 - \sigma_2$,$u = \mu_1 + \sigma_1$ and $v = \mu_2 + \sigma_2$.

Since T is log – concave function of triangular fuzzy random variable.

ThenP $\{bT_\alpha^L + (1\text{-}b)T_\alpha^U\} \geq P\,\{(T_\alpha^L)^b\}\ P\,\{\{(T_\alpha^U)^{1-b}\}$

$$P\,\{(bT_\alpha^L-(\alpha-1)b\sigma_1-b\mu_1) \geq 0 \vee ((1-b)T_\alpha^U + (\alpha-1)(1-b)\sigma_1-(1-b)\mu_1) \leq 0\}$$
$$\geq P\,\{\big((T_\alpha^L-(\alpha-1)\sigma_1-\mu_1) \geq 0\ \big)^b\}\ P\,\{\big((T_\alpha^U + (\alpha-1)\sigma_1-\mu_1) \leq 0\ \big)^{1-b}\},\qquad 0 \leq b \leq 1.$$

Now, (LCFLR) can be written as

$$\frac{P\{(bT_\alpha^L-(\alpha-1)b\sigma_1 - b\mu_1) \geq 0\ \vee ((1-b)T_\alpha^U + (\alpha-1)(1-b)\sigma_1-(1-b)\mu_1)) \leq 0\}}{.\,P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1-\mu_1) \leq 0\}} \leq \frac{P\{(bT_\alpha^L-(\alpha-1)b\sigma_2 - b\mu_2) \geq 0\ \vee ((1-b)T_\alpha^U +\ (\alpha-1)(1-b)\sigma_2-(1-b)\mu_2)) \leq 0\}}{.\,P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2) \geq 0 \vee (R_\alpha^U\ +\ (\alpha-1)\sigma_2-\mu_2) \leq 0\}} \tag{3.2.1}$$

And it can also be written as

$$\frac{P\{\big((T_\alpha^L-(\alpha-1)\sigma_1-\mu_1) \geq 0\ \big)^b\}\,P\{\big((T_\alpha^U + (\alpha-1)\sigma_1-\mu_1) \leq 0\ \big)^{1-b}\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0\ \vee\ (R_\alpha^U + (\alpha-1)\sigma_1-\mu_1) \leq 0\}} \leq \frac{P\{\big((T_\alpha^L-(\alpha-1)\sigma_2-\mu_2) \geq 0\ \big)^b\}\,P\{\big((T_\alpha^U + (\alpha-1)\sigma_2-\mu_2) \leq 0\ \big)^{1-b}\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2) \geq 0\ \vee\ (R_\alpha^U +\ (\alpha-1)\sigma_2-\mu_2) \leq 0\}}$$

(3.2.2)

Here, LCFLR of the equation (3.2.1) $\geq$ equation (3.2.2), otherwise, is called LCONFLROTFN. It will write R $\leq^{LCFLR}$ T andR $\leq^{LCONFLR}$ T respectively. Here, the constants b and (1-b) are the left and right part of the triangular fuzzy number.In this chapter, we use the equation (3.2.1) only.

3.3. Proportional Fuzzy Likelihood Ratio Order of Triangular Fuzzy Random Variables
1.3.1 Proportional Fuzzy Likelihood Ratio Order
Definition: 1.3.1.1

If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1,σ_2 respectively. Then R is said to be proportional fuzzy likelihood ratio order of the triangular fuzzy number (PFLR) which is less than (or)equal to T. if whenever s $\leq$ t and u $\leq$ v.
Here, s = $\mu_1-\sigma_1$, t = $\mu_2-\sigma_2$, u = $\mu_1 + \sigma_1$, v = $\mu_2 + \sigma_2$and $\lambda < 1$.

$$P\{(R^L_\alpha-(\alpha-1)\sigma_2-\mu_2) \geq 0 \vee (R^U_\alpha+(\alpha-1)\sigma_2-\mu_2) \leq 0\}$$
$$P\{(\lambda T^L_\alpha-(\alpha-1)\lambda\sigma_1-\lambda\mu_1) \geq 0 \vee (\lambda T^U_\alpha+(\alpha-1)\lambda\sigma_1-\lambda\mu_1) \leq 0\}$$
$$\leq \quad P\{(R^L_\alpha-(\alpha-1)\sigma_1-\mu_1) \geq 0 \vee (R^U_\alpha+(\alpha-1)\sigma_1-\mu_1) \leq 0\}$$
$$P\{(\lambda T^L_\alpha-(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \geq 0 \vee (\lambda T^U_\alpha+(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \leq 0\}$$

It can be written as

$$\frac{P\{(\lambda T^L_\alpha-(\alpha-1)\lambda\sigma_1-\lambda\mu_1) \geq 0 \ \vee \ (\lambda T^U_\alpha+(\alpha-1)\lambda\sigma_1-\lambda\mu_1) \leq 0\}}{P\{(R^L_\alpha-(\alpha-1)\sigma_1-\mu_1) \geq 0 \ \vee (R^U_\alpha+(\alpha-1)\sigma_1-\mu_1) \leq 0\}}$$
$$\leq \ \frac{P\{(\lambda T^L_\alpha-(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \geq 0 \ \vee \ (\lambda T^U_\alpha+(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \leq 0\}}{P\{(R^L_\alpha-(\alpha-1)\sigma_2-\mu_2) \geq 0 \ \vee \ (R^U_\alpha+(\alpha-1)\sigma_2-\mu_2) \leq 0\}} \quad (3.3.1.1)$$

Here, the RHS is increasing in triangular fuzzy random variable for all λ in (0, 1).
We will write R $\leq^{PFLR}$ T.

Theorem: 1.3.1.2
If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1,σ_2 respectively.
If R $\leq^{PFLROTFN}$ T. Then$\mu_R \leq \mu_T$.
Proof:
If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1,σ_2 respectively.
Whenever,

$$P\{\mu_1-\sigma_1 \leq R \leq \mu_1+\sigma_1\} = P\{(R^L_\alpha-(\alpha-1)\sigma_1-\mu_1) \geq 0 \ \vee (R^U_\alpha+(\alpha-1)\sigma_1-\mu_1) \leq 0\}$$
$$= P\{(R^L_\alpha-\alpha\sigma_1-(\mu_1-\sigma_1)) \geq 0 \ \vee (R^U_\alpha+\alpha\sigma_1-(\mu_1+\sigma_1)) \leq 0\} \quad (3.3.1.2)$$
$$P\{\mu_1-\sigma_1 \leq T \leq \mu_1+\sigma_1\} = P\{(T^L_\alpha-(\alpha-1)\sigma_1-\mu_1) \geq 0 \ \vee (T^U_\alpha+(\alpha-1)\sigma_1-\mu_1) \leq 0\}$$
$$= P\{(T^L_\alpha-\alpha\sigma_1-(\mu_1-\sigma_1)) \geq 0 \ \vee \ (T^U_\alpha+\alpha\sigma_1-(\mu_1+\sigma_1)) \leq 0\} \quad (3.3.1.3)$$
$$P\{\mu_2-\sigma_2 \leq R \leq \mu_2+\sigma_2\} = P\{(R^L_\alpha-(\alpha-1)\sigma_2-\mu_2) \geq 0 \ \vee \ (R^U_\alpha+(\alpha-1)\sigma_2-\mu_2) \leq 0\}$$
$$= P\{(R^L_\alpha-\alpha\sigma_2-(\mu_2-\sigma_2)) \geq 0 \vee (R^U_\alpha+\alpha\sigma_2-(\mu_2+\sigma_2)) \leq 0\} \quad (3.3.1.4)$$
$$P\{\mu_2-\sigma_2 \leq T \leq \mu_2+\sigma_2\} = P\{(T^L_\alpha-(\alpha-1)\sigma_2-\mu_2) \geq 0 \vee \ (T^U_\alpha+(\alpha-1)\sigma_2-\mu_2) \leq 0\}$$
$$= P\{(T^L_\alpha-\alpha\sigma_2-(\mu_2-\sigma_2)) \geq 0 \ \vee (T^U_\alpha+\alpha\sigma_2-(\mu_2+\sigma_2)) \leq 0\} \quad (3.3.1.5)$$
$$P\{\mu_2-\sigma_2 \leq \lambda T \leq \mu_2+\sigma_2\} = P\{(\lambda T^L_\alpha-(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \geq 0 \vee (\lambda T^U_\alpha+(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \leq 0\}$$
$$= P\{(\lambda T^L_\alpha-\alpha\lambda\sigma_2-\lambda(\mu_2-\sigma_2)) \geq 0 \ \vee (\lambda T^U_\alpha+\alpha\lambda\sigma_2-\lambda(\mu_2+\sigma_2)) \leq 0\} \quad (3.3.1.6)$$
$$P\{\mu_2-\sigma_2 \leq \frac{T}{\lambda} \leq \mu_2+\sigma_2\} = P\left\{\left(\frac{T^L_\alpha}{a}-\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right) \geq \ 0 \ \vee \ \left(\frac{T^U_\alpha}{a}+\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right) \ \leq \ 0\right\}$$
$$= P\left\{\left(\frac{T^L_\alpha}{a}-\frac{\alpha\sigma_2}{\lambda}-\left(\frac{\mu_2-\sigma_2}{\lambda}\right)\right) \geq \ 0 \ \vee \ \left(\frac{T^U_\alpha}{a}+\frac{\alpha\sigma_2}{\lambda}-\left(\frac{\mu_2-\sigma_2}{\lambda}\right)\right) \ \leq \ 0\right\} \quad (3.3.1.7)$$

Let T_λ be the triangular fuzzy number of$\frac{T}{\lambda}$. Suppose, by contradiction, that $\mu_R > \mu_T$.
Since, P $\{\mu-\sigma \leq \lambda T \leq \ \mu+\sigma\}$ = $\frac{1}{\lambda}$ P$\{\mu-\sigma \leq \frac{T}{\lambda} \leq \ \mu+\sigma\}$, Where $\lambda = \frac{1}{a}$ <1, a>1.
In triangular fuzzy random variable, this equation can be written as follows:
(1) If T is a triangular fuzzy random variable. Then

$$P\{(\lambda T^L_\alpha-(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \geq \ 0 \vee \ (\lambda T^U_\alpha+(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \leq 0\}$$
$$= \ \frac{1}{\lambda} \ P\left\{\left(\frac{T^L_\alpha}{a}-\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right) \geq \ 0 \ \vee \ \left(\frac{T^U_\alpha}{a}+\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right) \ \leq \ 0\right\}$$

(2) If R is a triangular fuzzy random variable. Then

$$P\{(\lambda R^L_\alpha-(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \geq \ 0 \vee(\lambda R^U_\alpha+(\alpha-1)\lambda\sigma_2-\lambda\mu_2) \leq 0\}$$
$$= \frac{1}{\lambda} \ P\left\{\left(\frac{R^L_\alpha}{a}-\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right) \geq \ 0 \ \vee \ \left(\frac{R^U_\alpha}{a}+\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right) \ \leq \ 0\right\} \quad (3.3.1.8)$$

Where $\lambda = \frac{1}{a} < 1, a > 1$.It follows from the assumption that $\frac{P\{\lambda\mu-\lambda\sigma \le \lambda T \le \lambda\mu+\lambda\sigma\}}{P\{\mu-\sigma \le R \le \mu+\sigma\}}$ is increasing in triangular fuzzy random variable for all λ in (0, 1). Hence, S (T_λ- R) = 1 for each λ in (0, 1). Here, S (T_λ- R) means that the number of sign changes of the functions$T_\lambda$and R. (i.e.,) R and T_λ are stochastically ordered for eachλ in (0, 1). In particular, by taking $\lambda = \frac{\mu_T}{\mu_R} < 1$. It follows that the triangular fuzzy random variables R and $\frac{\mu_R\, T}{\mu_T}$ are stochastically ordered. Since R and $\frac{\mu_R\, T}{\mu_T}$ have the same mean, ordinary stochastic order is possible. If they have the same distribution. This contradicts (3.3.8) and hence $\mu_R \le \mu_T$ holds.

Theorem: 1.3.1.3:
If R $\le^{PFLR}$ T, then $\mu_1 - \sigma_1 \le \mu_2 - \sigma_2and\mu_1 + \sigma_1 \le \mu_2 + \sigma_2$.
Proof:
Suppose$\mu_1 - \sigma_1 > \mu_2 - \sigma_2$. Let $\in_1$ and $\in_2$ be such that $\mu_2 - \sigma_2 < \in_1 < \mu_1 - \sigma_1 < \in_2 < \min\{\mu_1 + \sigma_1, \mu_2 + \sigma_2\}$ and let$\lambda \in (0,1)$ such that $\mu_2 - \sigma_2 < \lambda \in_1 < \mu_1 - \sigma_1 < \lambda \in_2 < \min\{\mu_1 + \sigma_1, \mu_2 + \sigma_2\}$. By definition of PFLR under thegiven condition, we get,

$$\frac{P\left\{\left(\lambda T_\alpha^L - \alpha\lambda\sigma_2 - \lambda\left(\mu_2 - \sigma_2\right)\right) \ge 0 \vee \left(\lambda T_\alpha^U + \alpha\lambda\sigma_2 - \lambda\left(\mu_2 + \sigma_2\right)\right) \le 0\right\}}{P\left\{\left(R_\alpha^L - \alpha\sigma_2 - \left(\mu_2 - \sigma_2\right)\right) \ge 0 \vee \left(R_\alpha^U + \alpha\sigma_2 - \left(\mu_2 + \sigma_2\right)\right) \le 0\right\}} \le \frac{P\left\{\left(\lambda T_\alpha^L - \alpha\lambda\sigma_1 - \lambda\left(\mu_1 - \sigma_1\right)\right) \ge 0 \vee \left(\lambda T_\alpha^U + \alpha\lambda\sigma_1 - \lambda\left(\mu_1 + \sigma_1\right)\right) \le 0\right\}}{P\left\{\left(R_\alpha^L - \alpha\sigma_1 - \left(\mu_1 - \sigma_1\right)\right) \ge 0 \vee \left(R_\alpha^U + \alpha\sigma_1 - \left(\mu_1 + \sigma_1\right)\right) \le 0\right\}}$$

$$\therefore \frac{P\{(\lambda T_\alpha^L - \alpha\lambda\sigma_2 - \lambda^2 \in_1) \ge 0 \vee (\lambda T_\alpha^U + \alpha\lambda\sigma_2 - \lambda(\mu_2 + \sigma_2)) \le 0\}}{P\{(R_\alpha^L - \alpha\sigma_2 - (\mu_2 - \sigma_2)) \ge 0 \vee (R_\alpha^U + \alpha\sigma_2 - (\mu_2 + \sigma_2)) \le 0\}} \le \frac{P\{(\lambda T_\alpha^L - \alpha\lambda\sigma_1 - \lambda^2 \in_2) \ge 0 \vee (\lambda T_\alpha^U + \alpha\lambda\sigma_1 - \lambda(\mu_1 + \sigma_1)) \le 0\}}{P\{(R_\alpha^L - \alpha\sigma_1 - (\mu_1 - \sigma_1)) \ge 0 \vee (R_\alpha^U + \alpha\sigma_1 - (\mu_1 + \sigma_1)) \le 0\}}$$

Which is a contradiction to the definition of PFLR. Therefore, we must have $\mu_1 - \sigma_1 \le \mu_2 - \sigma_2$. Similarly, it can be shown that $\mu_1 + \sigma_1 \le \mu_2 + \sigma_2$.

Theorem: 1.3.1.4
If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviationsσ_1,σ_2 respectively. Satisfy R$\le^{PFLR}$ T if and only if R $\le^{FLR}$ aT,a >1.

Proof: Since R $\le^{PFLR}$ T

$$\Leftrightarrow \frac{P\{(\lambda T_\alpha^L - (\alpha-1)\lambda\sigma_1 - \lambda\mu_1) \ge 0 \vee (\lambda T_\alpha^U + (\alpha-1)\lambda\sigma_1 - \lambda\mu_1) \le 0\}}{P\{(R_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \ge 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \le 0\}} \le \frac{P\{(\lambda T_\alpha^L - (\alpha-1)\lambda\sigma_2 - \lambda\mu_2) \ge 0 \vee (\lambda T_\alpha^U + (\alpha-1)\lambda\sigma_2 - \lambda\mu_2) \le 0\}}{P\{(R_\alpha^L - (\alpha-1)\sigma_2 - \mu_2) \ge 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \le 0\}}$$

Where the RHS is increasing in triangular fuzzy random variable for all λ in (0, 1).
Here, $\lambda = \frac{1}{a} < 1$, a>1. We get,

$$\Leftrightarrow \frac{\left[P\{(\frac{T_\alpha^L}{a} - (\alpha-1)\frac{\sigma_1}{a} - \frac{\mu_1}{a}) \ge 0 \vee (\frac{T_\alpha^U}{a} + (\alpha-1)\frac{\sigma_1}{a} - \frac{\mu_1}{a}) \le 0\}\right]}{P\{(R_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \ge 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \le 0\}} \le \frac{\left[P\{(\frac{T_\alpha^L}{a} - (\alpha-1)\frac{\sigma_2}{a} - \frac{\mu_2}{a}) \ge 0 \vee (\frac{T_\alpha^U}{a} + (\alpha-1)\frac{\sigma_2}{a} - \frac{\mu_2}{a}) \le 0\}\right]}{P\{(R_\alpha^L - (\alpha-1)\sigma_2 - \mu_2) \ge 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \le 0\}}$$

By using equation (3.3.1.8), we get

$$\Leftrightarrow \frac{P\{(aT_\alpha^L - (\alpha-1)\, a\sigma_1 - a\mu_1) \ge 0 \vee (aT_\alpha^U + (\alpha-1)\, a\sigma_1 - a\mu_1) \le 0\}}{P\{(R_\alpha^L - (\alpha-1)\,\sigma_1 - \mu_1) \ge 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \le 0\}} \le \frac{P\{(aT_\alpha^L - (\alpha-1)\, a\sigma_2 - a\mu_2) \ge 0 \vee (aT_\alpha^U + (\alpha-1)\, a\sigma_2 - a\mu_2) \le 0\}}{P\{(R_\alpha^L - (\alpha-1)\,\sigma_2 - \mu_2) \ge 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \le 0\}}$$

Here, the RHS is increasing in triangular fuzzy random variable for all λ in (0, 1). Which implies that R $\le^{FLR}$ aT.

Theorem: 1.3.1.5

Let T be a triangular fuzzy random variable with mean μ_1 and standard deviation σ_1 and T is log – concave. Then T $\leq^{FLR}$ aT.

Proof: Since T is log – concave function of triangular fuzzy random variable with mean μ_1 and standard deviation σ_1.Then $P\{cT_\alpha^L + (1\text{-}c)T_\alpha^U\} \geq P\{(T_\alpha^L)^c\}\, P\{(T_\alpha^U)^{1-c}\}$

$$P\{(cT_\alpha^L-(\alpha-1)c\sigma_1-c\mu_1)\geq 0 \vee ((1-c)T_\alpha^U+(\alpha-1)(1-c)\sigma_1-(1-c)\mu_1)\leq 0\}$$
$$\geq P\{\left((T_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0\right)^c\}\, P\{\left((T_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\right)^{1-c}\} \quad (3.3.1.9)$$

Here, the constants c and (1-c) are the left and right part of the triangular fuzzy number.andaT is also a log – concave function of triangular fuzzy random variable with mean a$\mu_1 = \mu_2$ and Standard deviation a$\sigma_1 = \sigma_2$.

We get

$$P\{(abT_\alpha^L-(\alpha-1)b\sigma_2-b\mu_2)\geq 0 \vee (a(1-b)T_\alpha^U+(\alpha-1)(1-b)\sigma_2-(1-b)\mu_2)\leq 0\}\geq$$
$$P\{\left((aT_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0\right)^b\}\, P\{\left((aT_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\right)^{1-b}\} \quad (3.3.1.10)$$

Here, the constants b and (1-b) are the left and right part of the triangular fuzzy number.

By using the equation (3.3.1.1) in the equation (3.3.1.9) and (3.10). We get,

$$\frac{P\{(abT_\alpha^L-(\alpha-1)b\sigma_1-b\mu_1)\geq 0 \vee (a(1-b)T_\alpha^U+(\alpha-1)(1-b)\sigma_1-(1-b)\mu_1)\leq 0\}}{P\{(cT_\alpha^L-(\alpha-1)c\sigma_1-c\mu_1)\geq 0 \vee ((1-c)T_\alpha^U+(\alpha-1)(1-c)\sigma_1-(1-c)\mu_1)\leq 0\}}$$
$$\leq \frac{P\{(abT_\alpha^L-(\alpha-1)b\sigma_2-b\mu_2)\geq 0 \vee (a(1-b)T_\alpha^U+(\alpha-1)(1-b)\sigma_2-(1-b)\mu_2)\leq 0\}}{P\{(cT_\alpha^L-(\alpha-1)c\sigma_2-c\mu_2)\geq 0 \vee ((1-c)T_\alpha^U+(\alpha-1)(1-c)\sigma_2-(1-c)\mu_2)\leq 0\}}$$

Which is implies that T $\leq^{FLR}$ aT.

Theorem: 1.3.1.6

If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1,σ_2 respectively. Suppose that T has a log – concave function. Then R $\leq^{FLR}$ T

$\Rightarrow$ R $\leq^{PFLR}$ T.

Proof: Since R $\leq^{FLR}$ T and T has a log – concave function. Then

$$\frac{P\{(bT_\alpha^L-(\alpha-1)b\sigma_1-b\mu_1)\geq 0 \vee ((1-b)T_\alpha^U+(\alpha-1)(1-b)\sigma_1-(1-b)\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}}$$
$$\leq \frac{P\{(bT_\alpha^L-(\alpha-1)b\sigma_2-b\mu_2)\geq 0 \vee ((1-b)T_\alpha^U+(\alpha-1)(1-b)\sigma_2-(1-b)\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Here, b and (1 – b) are the left and right part of the triangular fuzzy number and $0\leq b\leq 1$.

Now, we use the relationship between λ and b ($\lambda < b$, $\lambda \neq 0$). We get

$$\frac{P\{(\lambda T_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \vee ((1-\lambda)T_\alpha^U+(\alpha-1)(1-\lambda)\sigma_1-(1-\lambda)\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}}$$
$$\leq \frac{P\{(\lambda T_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \vee ((1-\lambda)T_\alpha^U+(\alpha-1)(1-\lambda)\sigma_2-(1-\lambda)\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Which implies that R $\leq^{PFLR}$ T.

3.3.2 Increasing and Decreasing Proportional Fuzzy Likelihood Ratio Order

Definition: 1.3.2.1

Let R be a triangular fuzzy random variable with mean μ and standard deviation σ. Then R is said to be increasing proportional fuzzy likelihood ratio order of triangular fuzzy number (IPFLROTFN) R $\leq^{IPFLROTFN}$R. If

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}}$$
$$\leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

the RHS is increasing in triangular fuzzy random variable for all λ in (0,1).

By theorem (3.3.1.1), we have R $\leq^{IPFLR}$R with means $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$ respectively. It will be said that R is decreasing proportional fuzzy likelihood ratio order of the triangular fuzzy number (DPFLR) R $\leq^{DPFLR}$ R. If

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}}$$
$$\leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

the RHS is decreasing in triangular fuzzy random variable for all λ in (0,1).

Theorem: 1.3.2.2:

Let R be a triangular fuzzy random variable with mean μ_1 and standard deviation σ_1. The following conditions are equivalent:1) R $\leq^{IPFLR}$R 2) R $\leq^{FLR}$ aR, $\forall$ a > 1. 3) R $\leq^{PFLR}$ R

Proof: Since R$\leq^{IPFLR}$ R

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \vee (\lambda R_\alpha^L+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

the RHS is increasing in triangular fuzzy random variable for all λ in (0,1).

By using equation (3.3.1.8), we get

$$\frac{P\{(\frac{R_\alpha^L}{\lambda}- - (\alpha-1)\frac{\sigma_1}{\lambda}-\frac{\mu_1}{\lambda})\geq 0 \vee (\frac{R_\alpha^U}{\lambda}+(\alpha-1)\frac{\sigma_1}{\lambda}-\frac{\mu_1}{\lambda})\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\frac{R_\alpha^L}{\lambda}-(\alpha-1)\frac{\sigma_2}{\lambda}\frac{\mu_2}{\lambda})\geq 0 \vee (\frac{R_\alpha^U}{\lambda}+(\alpha-1)\frac{\sigma_2}{\lambda}\frac{\mu_2}{\lambda})\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Put $a=\frac{1}{\lambda}$, a > 1, we get

$$\frac{P\{(aR_\alpha^L-(\alpha-1)a\sigma_1-a\mu_1)\geq 0 \vee (aR_\alpha^U+(\alpha-1)a\sigma_1-a\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(aR_\alpha^L-(\alpha-1)a\sigma_2-a\mu_2)\geq 0 \vee (aR_\alpha^U+(\alpha-1)a\sigma_2-a\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Which implies that R $\leq^{FLRO}$ aR, $\forall$ a > 1. Hence, (1) $\Rightarrow$ (2)

2) Since R $\leq^{FLR}$ aR, $\forall$ a > 1.

$$\frac{P\{(aR_\alpha^L-(\alpha-1)a\sigma_1-a\mu_1)\geq 0 \vee (aR_\alpha^U+(\alpha-1)a\sigma_1-a\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(aR_\alpha^L-(\alpha-1)a\sigma_2-a\mu_2)\geq 0 \vee (aR_\alpha^U+(\alpha-1)a\sigma_2-a\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Here, the RHS is increasing triangular fuzzy random variable for all a >1.

Put $a=\frac{1}{\lambda}$, a > 1, we get

$$\frac{P\{(\frac{R_\alpha^L}{\lambda}- - (\alpha-1)\frac{\sigma_1}{\lambda}-\frac{\mu_1}{\lambda})\geq 0 \vee (\frac{R_\alpha^U}{\lambda}+(\alpha-1)\frac{\sigma_1}{\lambda}-\frac{\mu_1}{\lambda})\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\left\{\left(\frac{R_\alpha^L}{\lambda}-\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right)\geq 0 \vee \left(\frac{R_\alpha^U}{\lambda}+\frac{(\alpha-1)\sigma_2}{\lambda}-\frac{\mu_2}{\lambda}\right)\leq 0\right\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

By equation (3.3.1.8),

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Where the RHS is increasing in triangular fuzzy random variable for all $\lambda < 1$.

Which implies that R $\leq^{PFLR}$ R. Hence, (2) $\Rightarrow$ (3).

3) Since R $\leq^{PFLR}$ R.

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \vee (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \vee (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Here, the RHS is increasing in triangular fuzzy random variable for all $\lambda < 1$ and $\lambda = \frac{1}{a}$, a > 1. Which implies that (IPFLR).Hence, R $\leq^{PFLR}$ R $\Rightarrow$R $\leq^{IPFLR}$ R.Hence, (3) $\Rightarrow$ (1).

Theorem: 1.3.2.3

If R and T are triangular fuzzy random variables with means μ_1, μ_2 and standard deviations σ_1,σ_2 respectively. If R $\leq^{FLR}$ T and T is IPFLR. Then R $\leq^{PFLR}$ T.

3.16. Proof: Since R $\leq^{FLR}$ T

$$\frac{P\left\{\left(T_\alpha^L-(\alpha-1)\sigma_1-\mu_1\right)\geq 0 \ \vee\left(T_\alpha^U+(\alpha-1)\sigma_1-\mu_1\right)\leq 0\right\}}{P\left\{\left(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1\right)\geq 0 \ \vee\left(R_\alpha^U+(\alpha-1)\sigma_1-\mu_1\right)\leq 0\right\}} \leq \frac{P\left\{\left(T_\alpha^L-(\alpha-1)\sigma_2-\mu_2\right)\geq 0 \ \vee\left(T_\alpha^U+(\alpha-1)\sigma_2-\mu_2\right)\leq 0\right\}}{P\left\{\left(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2\right)\geq 0 \ \vee\left(R_\alpha^U+(\alpha-1)\sigma_2-\mu_2\right)\leq 0\right\}}$$

And if T is IPFLR. Then

$$\frac{P\{(\lambda T_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \ \vee \ (\lambda T_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(T_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \ \vee (T_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\left\{\left(\lambda T_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2\right)\geq 0 \ \vee\left(\lambda T_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2\right)\leq 0\right\}}{P\left\{\left(T_\alpha^L-(\alpha-1)\sigma_2-\mu_2\right)\geq 0 \ \vee\left(T_\alpha^U+(\alpha-1)\sigma_2-\mu_2\right)\leq 0\right\}}$$

Multiply the above two inequalities, we get

$$\frac{P\{(\lambda T_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \ \vee \ (\lambda T_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \ \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\lambda T_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \ \vee \ (\lambda T_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \ \vee \ (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Which implies that R $\leq^{PFLR}$ T.

Theorem: 1.3.2.4

Let R be a triangular fuzzy random variable with mean μ and standard deviation σ. Then R is IPFLR (DPFLR) if and only if R is log – concave (log – convex).

Proof: We will prove the result for the DPFLR case; the IPFLR case can be proven in the same way.Suppose that the triangular fuzzy random variable T is log – convex. Then

$$P\{bT_\alpha^L+(1\text{-}b)T_\alpha^U\}\leq P\{\left(T_\alpha^L\right)^b\}\ P\{\{\left(T_\alpha^U\right)^{1-b}\},\quad (0\leq b\leq 1)$$

$$P\{(bT_\alpha^L-(\alpha-1)b\sigma_1-b\mu_1)\geq 0 \vee ((1-b)T_\alpha^U+(\alpha-1)(1-b)\sigma_1-(1-b)\mu_1)\leq 0\}$$
$$\leq \ P\{\left((T_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0\ \right)^b\}\ P\{\left((T_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\ \right)^{1-b}\}$$

Here, the triangular fuzzy number (a_1, a_2, a_3) have the equal distance from the mean ($a_2 = \mu$). Therefore, the middle point

of the triangular fuzzy number is $\frac{b+(1-b)}{2} = 0.5 = \lambda$. By using definition (3.2.7) and definition (3.3.2.1), we get

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \ \vee \ (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \ \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \ \vee \ (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \ \vee \ (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Here, the RHS is decreasing in triangular fuzzy random variable for all λ in (0, 1).

This implies that R is DPFLR.

Conversely, assume that R is DPFLR and Let a >1, $\lambda = \frac{1}{a} < 1$.

$$\frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\geq 0 \ \vee \ (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_1-\lambda\mu_1)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_1-\mu_1)\geq 0 \ \vee (R_\alpha^U+(\alpha-1)\sigma_1-\mu_1)\leq 0\}} \leq \frac{P\{(\lambda R_\alpha^L-(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\geq 0 \ \vee \ (\lambda R_\alpha^U+(\alpha-1)\lambda\sigma_2-\lambda\mu_2)\leq 0\}}{P\{(R_\alpha^L-(\alpha-1)\sigma_2-\mu_2)\geq 0 \ \vee \ (R_\alpha^U+(\alpha-1)\sigma_2-\mu_2)\leq 0\}}$$

Here, the RHS is decreasing in triangular fuzzy random variable for all λ in (0, 1).

By equation (3.8),

$$\Rightarrow \frac{P\left\{\left(\frac{R_\alpha^L}{\lambda} - \frac{(\alpha-1)\sigma_1}{\lambda} - \frac{\mu_1}{\lambda}\right) \geq 0 \vee \left(\frac{R_\alpha^U}{\lambda} + \frac{(\alpha-1)\sigma_1}{\lambda} - \frac{\mu_1}{\lambda}\right) \leq 0\right\}}{P\{(R_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \leq 0\}}$$

$$\leq \frac{P\left\{\left(\frac{R_\alpha^L}{\lambda} - \frac{(\alpha-1)\sigma_2}{\lambda} - \frac{\mu_2}{\lambda}\right) \geq 0 \vee \left(\frac{R_\alpha^U}{\lambda} + \frac{(\alpha-1)\sigma_2}{\lambda} - \frac{\mu_2}{\lambda}\right) \leq 0\right\}}{P\{(R_\alpha^L - (\alpha-1)\,\sigma_2 - \mu_2) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \leq 0\}}$$

Then put $a = \frac{1}{\lambda}$, $a > 1$, we get

$$\frac{P\{(aR_\alpha^L - (\alpha-1)a\sigma_1 - a\mu_1) \geq 0 \vee (aR_\alpha^U + (\alpha-1)a\sigma_1 - a\mu_1) \leq 0\}}{P\{(R_\alpha^L - (\alpha-1)\,\sigma_1 - \mu_1) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \leq 0\}}$$

$$\leq \frac{P\{(aR_\alpha^L - (\alpha-1)a\sigma_2 - a\mu_2) \geq 0 \vee (aR_\alpha^U + (\alpha-1)a\sigma_2 - a\mu_2) \leq 0\}}{P\{(R_\alpha^L - (\alpha-1)\sigma_2 - \mu_2) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \leq 0\}}$$

Here, $a = 0.5 = \frac{b+(1-b)}{2}$ is the middle part of the triangular fuzzy random variable, b and (1-b) are the left and right part of the log – convex triangular fuzzy random variable. Hence,

$$\frac{P\{(bR_\alpha^L - (\alpha-1)b\sigma_1 - b\mu_1) \geq 0 \vee ((1-b)R_\alpha^U + (\alpha-1)(1-b)\sigma_1 - (1-b)\mu_1)) \leq 0\}}{P\{(R_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_1 - \mu_1) \leq 0\}}$$

$$\leq \frac{P\{(bR_\alpha^L - (\alpha-1)b\sigma_2 - b\mu_2) \geq 0 \vee ((1-b)R_\alpha^U + (\alpha-1)(1-b)\sigma_2 - (1-b)\mu_2)) \leq 0\}}{.\,P\{(R_\alpha^L - (\alpha-1)\sigma_2 - \mu_2) \geq 0 \vee (R_\alpha^U + (\alpha-1)\sigma_2 - \mu_2) \leq 0\}}$$

Which implies that $R \leq^{LCONFLR} R$ so that R is log – convex.
Hence, the proof.

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